Missouri Department of Transportation

Bridge Division

Bridge Design Manual

Section 1.3

Revised 08/19/2002

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Index of Distribution of Loads

1.3.1 Distribution of Dead Load

Composite Steel or Prestressed Concrete Structures

The dead load applied to the girders through the slab shall be:

Dead Load 1

Non-composite dead loads should be distributed to girders (stringers) on the basis of continuous spans over simple supports.

Dead Load 2

Composite loads shall be distributed equally to all girders. The following are all Dead Load 2 loads:

Barrier curb
Future wearing surface on slab
Sidewalks
Fences
Protective coatings and waterproofing on slab

Concrete Slab Bridges

Distribute entire dead load across full width of slab.

For longitudinal design, heavier portions of the slab may be considered as concentrated load for entry into the "Continuous Structure Analysis" computer program.

For transverse bent design, consider the dead load reaction at the bent to be a uniform load across entire length of the transverse beam.

1.3.2 Distribution of Live Load

Live loading to be distributed shall be the appropriate loading shown on the Design Layout.

Applying Live Load to Structure

Superstructure

For application of live load to superstructure, the lane width is considered 10 feet. Each design vehicle has wheel lines which are 6 feet apart and adjacent design vehicles must be separated by 4 feet.

Substructure

To produce the maximum stresses in the main carrying members of substructure elements, multiple lanes are to be loaded simultaneously. The lane width is 10 feet. Partial lanes are not to be considered. Due to the improbability of coincident maximum loading, a reduction factor is applied to the number of lanes. *This reduction however, is not applied in determining the distribution of loads to the stringers*.

AASHTO 3.12

Number of Lanes	Percent
one or two lanes	100
three lanes	90
four lanes or more	75

Distribution of Live Load to Beams and Girders

AASHTO 3.23

Moment Distribution

Moments due to live loads shall not be distributed longitudinally. Lateral distribution shall be determined from AASHTO Table 3.23.1 for interior stringers. Outside stringers distribute live load assuming the flooring to act as a simple span, except in the case of a span with a concrete floor supported by four or more stringers, then AASHTO 3.23.2.3.1.5 shall be applied. In no case shall an exterior stringer have less carrying capacity than an interior stringer.

Shear Distribution

As with live load moment, the reactions to the live load are not to be distributed longitudinally. Lateral distribution of live load shall be that produced by assuming the flooring to act as simply supported. Wheel lines shall be spaced on accordance with AASHTO 3.7.6 and shall be placed in a fashion which provides the most contribution to the girder under investigation, regardless of lane configuration. The shear distribution factor at bents shall be used to design bearings and bearing stiffeners.

Deflection Distribution

Deflection due to live loads shall not be distributed longitudinally. Lateral distribution shall be determined by averaging the moment distribution factor and the number of wheel lines divided by the number of girder lines for all girders. The number of wheel lines shall be based on 10 foot lanes. The reduction in load intensity (AASHTO Article 3.12.1) shall not be applied.

Deflection Distribution Factor = $\dfrac{\{\frac{2n}{N}\} + MDF}{2}$

Where: n = number of whole 10 foot lanes on the roadway;
N = number of girder lines;
MDF = Moment Distribution Factor.

Example: 38'-0" Roadway (Interior Girder), n=3, N=5, MDF=1.576

Deflection Distribution Factor = $\dfrac{\{\frac{2\ x\ 3lanes}{5\ girders}\} + 1.576}{2}$ =1.388

Live Load Distribution Factors for Standard Roadway Widths

Roadway Width	Number Girders	Girder Spacing	Exterior Girder Mom.	Exterior Girder Shear	Exterior Girder Defl.	Interior Girder Mom.	Interior Girder Shear	Interior Girder Defl.	(1)
26’-0”	4	7’-6”	1.277	1.133	1.139	1.364	1.667	1.182	1.071
28’-0”	4	8’-2”	1.352	1.204	1.176	1.485	1.776	1.243	1.167
30’-0”	4	8’-8”	1.405	1.308	1.453	1.576	1.846	1.538	1.238
32’-0”	4	9’-2”	1.457	1.400	1.479	1.667	1.909	1.584	1.310
36’-0”	5	8’-2”	1.352	1.184	1.276	1.485	1.776	1.343	1.167
38’-0”	5	8’-8”	1.405	1.231	1.303	1.576	1.846	1.388	1.238
40’-0”	5	9’-0”	1.440	1.333	1.520	1.636	1.889	1.618	1.286
44’-0”	5	9’-9”	1.515	1.487	1.558	1.773	1.974	1.687	1.393

(1) Use when checking interior girder moment cyclical loading Case I Fatigue for one lane loading.

Distribution of Live Load to Substructure

For substructure design the live load wheel lines shall be positioned on the slab to produce maximum moments and shears in the substructure. The wheel lines shall be distributed to the stringers on the basis of simple spans between stringers. The number of wheel lines used for substructure design shall be based on 10 foot lanes and shall not exceed the number of lanes times two with the appropriate percentage reduction for multiple lanes where applicable.

In computing these stresses generated by the lane loading, each 10 foot lane shall be considered a unit. Fractional units shall not be considered.

Distribution of Loads to Slabs

AASHTO 3.24.1

For simple spans, the span length shall be the distance center to center of supports but need not be greater than the clear distance plus the thickness of the slab. Slabs for girder and floor beam structures should be designed as supported on four sides.

AASHTO 3.24.6

For continuous spans on steel stringers or on thin flanged prestressed beams (top flange width to thickness ratios > 4.0), the span length shall be the distance between edges of top flanges plus one quarter of each top flange width. When the top flange width to thickness is < 4.0 the span distance shall be the clear span between edges of the top flanges.

AASHTO 3.24.2

When designing the slab for live load, the wheel line shall be placed 1 foot from the face of the barrier curb if it produces a greater moment.

Bending Moments in Slab on Girders

Main Reinforcement Perpendicular to Traffic

AASHTO 3.24.3.1

The load distributed to the stringers shall be

$\left(\frac{S+2}{32}\right)$P20 or P25 = Moment in foot-pounds per-foot width of slab.

Where
S = effective span length between girders in feet;
P20 or P25= wheel line load for HS20 or HS20 Modified design Truck in kips.

For slabs continuous over 3 or more supports, a continuity factor of 0.8 shall be applied.

Main Reinforcement Parallel to Traffic

AASHTO 3.24.3.2

This distribution may be applied to special structure types when its use is indicated.

Distribution of Live Load to Concrete Slab Bridges

AASHTO 3.24

Live load for transverse beam, column and pile cap design shall be applied as concentrated loads of one wheel line. The number of wheel lines used shall not exceed the number of lanes x 2 with the appropriate reduction where applicable.

AASHTO 3.24.3.2

For slab longitudinal reinforcement design, use live load moment distribution factor of 1/E for a one-foot strip slab with the appropriate percentage reduction.

E = 4' + 0.06S, E (max.) = 7'

where:

E = Width of slab in feet over which a wheel is distributed;
S = Effective span length in feet.

For slab deflection, use the following deflection factor for a one-foot strip slab without applying percentage reduction.

Deflection Factor = (Total number of wheel line) / (width of the slab)

See also Section 3.52, page 1.7-1 for modulus of elasticity of slab for deflection computation.

1.3.3 Frictional Resistance

The frictional resistance varies with different surfaces making contact. In the design of bearings, this resistance will alter how the longitudinal forces are distributed. The following table lists commonly encountered materials and their coefficients. These coefficients may be used to calculate the frictional resistance at each bent.

<table>
<tr><th colspan="4">Frictional Resistance of Expansion Bearings</th></tr>
<tr><th colspan="2">Bearing Type</th><th>Coef.</th><th>General Data</th></tr>
<tr><td colspan="2">Type C Bearing</td><td>0.14</td><td rowspan="10">Coef. of sliding friction steel to steel = 0.14

Coef. for pin and rocker type bearing =
$\frac{0.14\ (\text{Radius of pin})}{\text{Radius of Rocker}}$

Frictional Force = Reaction x Coef.</td></tr>
<tr><td colspan="2">6” Diameter Roller</td><td>0.01</td></tr>
<tr><td colspan="2">Type D Bearing</td><td></td></tr>
<tr><td>Pin Diameter</td><td>Rocker Radius</td><td></td></tr>
<tr><td>2”</td><td>6.5”</td><td>0.0216</td></tr>
<tr><td>2”</td><td>7”</td><td>0.0200</td></tr>
<tr><td>2”</td><td>7.5”</td><td>0.0187</td></tr>
<tr><td>2”</td><td>8”</td><td>0.0175</td></tr>
<tr><td>2”</td><td>10.5”</td><td>0.0133</td></tr>
<tr><td colspan="2">PTFE Bearing</td><td>0.0600</td></tr>
</table>

The design of a bent with one of the above expansion bearings will be based on the maximum amount of load the bearing can resist by static friction. When this static friction is overcome, the longitudinal forces are redistributed to the other bents.

The maximum static frictional force at a bent is equal to the sum of the forces in each of the bearings. The vertical reaction used to calculate this maximum static frictional force shall be Dead Loads only for all loading cases. Since the maximum longitudinal load that can be experienced by any of the above bearings is the maximum static frictional force, the effects of longitudinal wind and temperature can not be cumulative if their sum is greater than this maximum static frictional force.

Two conditions for the bents of the bridge are to be evaluated.

1. Consider the expansion bents to be fixed and the longitudinal loads distributed to all of the bents.

2. When the longitudinal loads at the expansion bearings are greater than the static frictional force, then the longitudinal force of the expansion bearings is equal to the dynamic frictional force. It is conservative to assume the dynamic frictional force to be zero causing all longitudinal loads to be distributed to the remaining bents.

1.3.4 Distribution of Longitudinal Wind

The total longitudinal wind load applied to the superstructure of a continuous series causes a small movement which deflects each support by an equal amount (see Figure 1 and Equation 1).

Equation 1 $\Delta 1 = \Delta 2 = - - - = \Delta i = - - - = \Delta n$

where Δi = The total deflections at Bent i
i = Bent (support) number
n = Total number of bents (supports)

The percentage of longitudinal wind load applied to any support can be found by calculating the support deflections in terms of **Pi**, and substituting them into the following equation:

Equation 2 LW = P1 + P2 + - - - + Pi + - - - + Pn

Where LW = Total longitudinal wind load (lbs)
Pi = Longitudinal wind load to Bent i, (support i)
i = Bent (support) number
n = Total number of bents (supports)

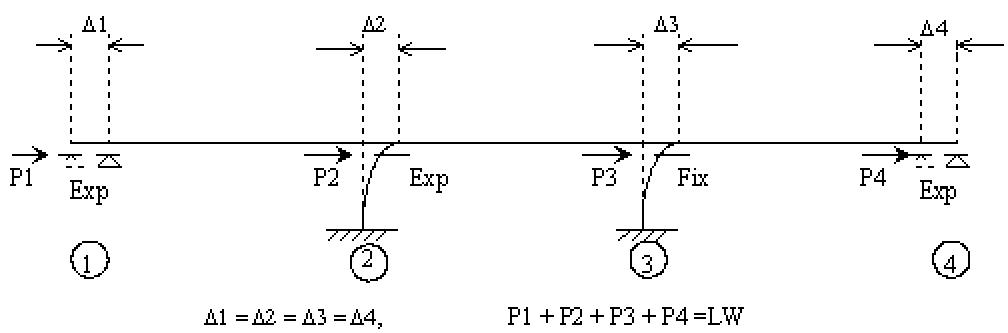
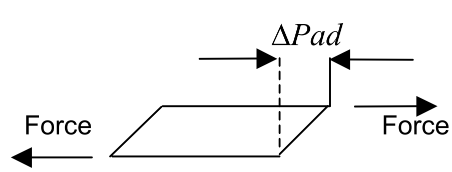


Figure 1

Elastomeric Bearings

Expansion bearing pads deflection, Δpads, can be calculated from the following equation (Use ΔPads = 0 if there are no expansion pads).

$$\Delta Pads = \frac{(Pi)(T)}{(L)(W)(G)(N)}$$



Where Pi = Longitudinal force to Bent i (lbs)
N = Total number of pads at Bent i
L = Length of pad (inch)
W = Width of pad (inch)
T = Total thickness of elastomer layers for pad (inch)
G = Shear Modulus (psi)

The shear modulus, G, varies with durometer, temperature, and time. To simulate this variance, the designer should run two sets of calculations, with G maximum and minimum.

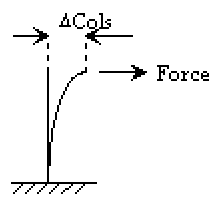
G (min) = 150 psi G (max) = 300 psi

Use 60 Durometer Pads.

Column

Column deflections, ΔCols, can be calculated from the following equation.

$$\Delta \text{ Cols} = \frac{(Pi)(H^3)}{3(E)(I)}$$



Where
Pi = Longitudinal force to Bent i (lb)
H = Bent height form point of fixity to top of beam (inch) (*)
I = Gross moment of inertia of bent (in^4), adjusted for skew (**)
E = Column modulus of elasticity (psi)

Total Deflection at Bent i: Δi

Δ i = ΔPads + ΔCols

If there are no Expansion pads, ΔPads = 0;
(i.e. fixed supports)

If the bent is nonflexible, ΔCols = 0.
(i.e. semi-deep abutments or Non-Integral end bents)

(*) For Pile Cap Intermediate Bents or Integral End Bents, use clear height plus Equivalent Cantilever Length defined in Seismic Design Section.

(**) See Section 1.3.6 and 1.3.7 for Gross Moment of Inertia for Column & Pile Cap Bents and Resultant Moment of Inertia respectively.

Example

3-Span continuous series (see page 4-1, Figure 1)

Bent No. 1 - Semi-Deep abutment with expansion bearing pads.
5-Pads: L x W x T = 8" x 17" x 4.5"
Assume the bent is nonflexible.

Bent No. 2 - Concrete column bent with expansion bearing pads.
5-Pads: L x W x T = 18" x 20" x 2.5"
Column H = 33.63' (from point of fixity to top of beam) = 403.56"
3-Columns: column diameter = 3.0 ft
I = 247,344 in^4 (moment of inertia of 3-columns)
E = 3.12 x10^6 psi (modulus of elasticity of column)

Bent No. 3 - Concrete column bent, fixed support.
Assume no pad deflection.
Column H = 33.63' = 403.56"
3-Columns: column diameter = 3.0 ft
I = 247,344 in^4 (moment of inertia of 3-columns)
E = 3.12 x10^6 psi (modulus of elasticity of column)

Bent No. 4 - Semi-Deep abutment with expansion bearing pads.
5-Pads: L x W x T = 7" x 20" x 2.25"
Assume the bent is nonflexible.

Find Percent of Longitudinal Wind with G (min) = 150 psi

$\Delta 1 = \Delta \text{Pads}$

$$\Delta 1 = \frac{(P1)(4.5")}{(8")(17")(5\ \ psi)} = 4.4117 \times 10^{-3}\ P1$$

$\Delta 2 = \Delta \text{Pads} + \Delta \text{Cols}$

$$\Delta 2 = \frac{(P2)(2.5")}{(18")(20")(5)(150\ \ psi)} + \frac{P2(403.56")^3}{3(3.12x10^6\ psi)(247{,}344\ \ in^4)}$$

$\Delta 2 = (0.9259+2.8389) \times 10^{-5}\ P2 = 3.7648 \times 10^{-5}\ P2$

$\Delta 3 = \Delta \text{Cols}$

$$\Delta 3 = \frac{(403.56")^3}{3(3.12x10^6\ psi)(247{,}344\ \ in^4)} = 2.8389 \times 10^{-5}\ P3$$

$\Delta 4 = \Delta \text{Pads}$

$$\Delta 4 = \frac{(P4)(2.25")}{(7")(20")(5)(150\ \ psi)} = 2.1428 \times 10^{-5}\ P4$$

Since $\Delta 1 = \Delta 2 = \Delta 3 = \Delta 4$ (**Equation 1**)

Find Percent of Longitudinal Wind with G (min) = 150 psi (Cont.)

Put Pi in terms of P1:

P1 = 1.0 x P1
P2 = (4.4117/3.7648) P1 = 1.1718 P1
P3 = (4.4117/2.8389) P1 = 1.5540 P1
P4 = (4.4117/2.1428) P1 = 2.0588 P1

Since LW = P1 + P2 + P3 + P4 (**Equation 2**)
then LW = P1 + 1.1718 P1 + 1.5540 P1 + 2.0558 P1
LW = 5.7846 P1

P1 = (1/5.7846) LW = 0.1729 LW or 17.3% of LW
P2 = 1.1718 x 0.1729 LW = 0.2026 LW or 20.3% of LW
P3 = 1.5540 x 0.1729 LW = 0.2687 LW or 26.9% of LW
P4 = 2.0558 x 0.1729 LW = 0.3554 LW or 35.5% of LW

Find Percent of Longitudinal Wind with G (max) = 300 psi

$\Delta 1 = 2.206 \times 10^{-5}$ P1
$\Delta 2 = 3.302 \times 10^{-5}$ P2
$\Delta 3 = 2.839 \times 10^{-5}$ P3
$\Delta 4 = 1.071 \times 10^{-5}$ P4

P1 = 1.000 P1
P2 = 0.668 P1
P3 = 0.777 P1
P4 = 2.059 P1

LW = P1 + 0.668 P1 + 0.777P1 + 2.059 P1 = 4.504 P1

P1 = 0.225 LW or 22.2% LW> 17.3% - USE 22.2%
P2 = 0.148 LW or 14.8% LW< 20.3% - USE 20.3%
P3 = 0.173 LW or 17.3% LW< 26.9% - USE 26.9%
P4 = 0.457 LW or 45.7% LW> 35.5% - USE 45.7%

(Note: Use the greater percentage from the calculations based on G = 150 psi or G = 300 psi.)

1.3.5 Distribution of Temperature Forces

The longitudinal temperature forces applied to a continuous series of girders causes an incremental movement which deflects the supporting columns and bearing pads. The longitudinal forces applied at each bent can be determined by the following procedure.

Example: ***4 Span Steel Structure (Figure1)***

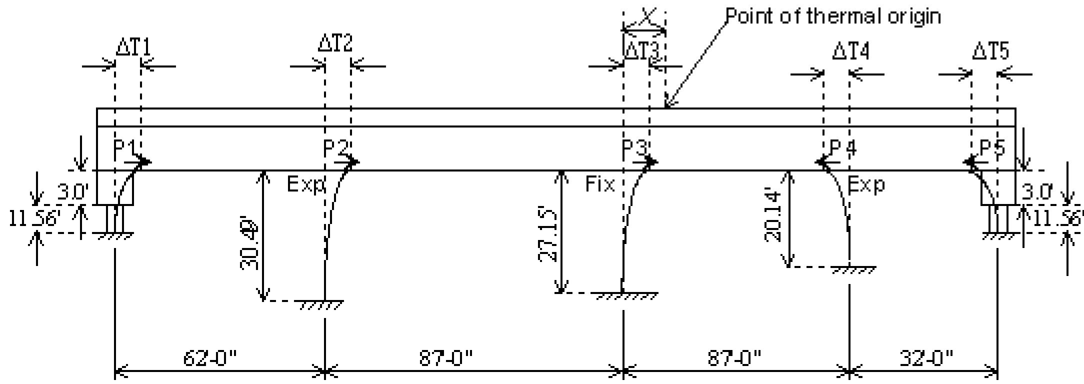


Figure 1

Thermal coefficients	Steel Coefficient = 0.0000065 ft/ft/^{O}F
	Concrete Coefficient = 0.000006 ft/ft/^{O}F
Temperature Range	Steel - 60^{O}F rise, 80^{O}F fall
	Concrete - 30^{O}F rise, 40^{O}F fall

Total Deflection at Bent i, ΔT_i (Temperature Fall)

ΔT1 = (62'+87'+ **X**) 80^{O}Fx0.0000065 ft/ft/^{O}Fx12"/ft = 0.9298"+ 0.00624 **X** (inch)

ΔT2 = (87'+ **X**) 80^{O}Fx0.0000065 ft/ft/^{O}Fx12"/ft = 0.5429"+ 0.00624 **X** (inch)

ΔT3 = (**X**) 80^{O}Fx0.0000065 ft/ft/^{O}Fx12"/ft = 0.00624 **X** (inch)

ΔT4 = (87'- **X**) 80^{O}Fx0.0000065 ft/ft/^{O}Fx12"/ft = 0.5429" - 0.00624 **X** (inch)

ΔT5 =(87'+32'- **X**) 80^{O}Fx0.0000065 ft/ft/^{O}Fx12"/ft = 0.7426" - 0.00624 **X** (inch)

Where **X** = Movement from Bent 3 to point of thermal origin (feet)

ΔT_i = Total deflection at Bent i (inch)

i = Bent number (support number)

Summation of Forces

P1 + P2 + P3 = P4 + P5

Where P_i = Longitudinal force at Bent i (lbs)

Deflection Computation at Bent i

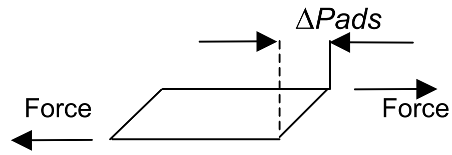
ΔT_i = ΔPads + ΔCols

ΔPads = 0, if there are no expansion pads (i.e. fixed supports)

ΔCols = 0, if the bent is nonflexible (i.e. semi-deep abutment or non-integral end bents)

Elastomeric Pad Deflection, Δ Pads

$$\Delta Pads = \frac{(Pi)(T)}{(L)(W)(G)(N)}$$



where ΔPads = Pads deflection (inch)
Pi = Longitudinal force at Bent i (lbs)
N = Number of pads at the bent
L = Length of pad (inch)
W = Width of pad (inch)
T = Total thickness of elastomer layers (inch)
G = Shear modulus (psi)

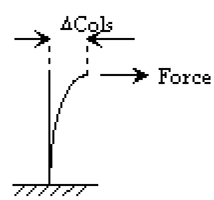
The shear modulus, G, varies with durometer, temperature, and time. To simulate this variance, the designer should run two sets of calculations. Use G maximum associated with temperature fall, and G minimum associated with temperature rise.

G (Min) = 150 psi use for temperature rise
G (Max) = 300 psi use for temperature fall

Use 60 Durometer Pads.

Column Deflections, Δ Cols

$$\Delta Cols = \frac{(Pi)(H^3)}{3(E)(I)}$$



where ΔCols = Deflection (inch)
Pi = Longitudinal temperature force at Bent i (lbs)
H = Bent height from point of fixity to top of beam (inch) (*)
I = Moment of inertia of bent (in^4), adjust for skew; (**)
E = Column modulus of elasticity (psi)

(*) For Pile Cap Intermediate Bents or Integral End Bents, use clear height plus Equivalent Cantilever Length defined in Seismic Design Section.

(**) See Section 1.3.6 and 1.3.7 for Gross and Resultant Moment of Initial for columns and pile cap bents respectively.

Distribution of Longitudinal Temperature Load to Bents

Bent No. 1 - Integral end bent with 5 HP10x42 piles

$I_{y\text{-}y}$ = 359 in^4, H=11.56'+3' = 14.56' =174.72", Es = 29x10^6 psi

$$\Delta T1 = \Delta Cols = \frac{P1(174.72")^3}{3(29x10^6\ psi)(359\ \ in^4)} = 17.077x10^{-5}P1$$

Bent No. 2 - Concrete column bent with expansion bearing pads

6 pads: LxWxT = 18" x 12" x 2", G = 300 psi, Ec = 3.3x10^6 psi
2 columns: column diameter = 3.0', I = 164896 in^4, H=30.49' =365.88"

$$\Delta Cols = \frac{P2(365.88")^3}{3(3.3x10^6\ psi)(164{,}896\ \ in^4)} = 3.000x10^{-5}P2$$

$$\Delta Pads = \frac{P2(2")}{(18")(12")(6)(300\ \ psi)} = 0.514x10^{-5}P2$$

ΔT2 = ΔPads + ΔCols = 3.514 x 10^{-5} P2

Bent No. 3 - Concrete column bent, fixed support

2 columns: column diameter = 3.0', I = 164896 in^4, H=27.15' = 325.8"

$$\Delta T3 = \Delta Cols = \frac{P3(325.8")^3}{3(3.3x10^6\ psi)(164{,}896\ \ in^4)} = 2.118x10^{-5}P3$$

Bent No. 4 - Concrete column bent with expansion bearings

6 pads: LxWxT = 18" x 12" x 2", G = 300 psi
2 columns: column diameter = 3.0', I = 164896 in^4, H = 20.14' = 241.68"

$$\Delta Cols = \frac{P4(241.68")^3}{3(3.3x10^6\ psi)(164{,}896\ \ in^4)} = 0.865x10^{-5}P4$$

$$\Delta Pads = \frac{P4(2")}{(18")(12")(6)(300\ \ psi)} = 0.514x10^{-5}P4$$

ΔT4 = ΔPads + ΔCols = 1.379 x 10^{-5} P4

Bent No. 5 - Integral end bent with 5 HP10x42 piles

$I_{y\text{-}y}$ = 359 in^4, H = 11.56'+3' = 14.56' =174.72"

$$\Delta T5 = \Delta Cols = \frac{P5(174.72")^3}{3(29x10^6\ psi)(359\ \ in^4)} = 17.077x10^{-5}P5$$

Summation of Forces

P1 + P2 + P3 = P4 + P5

where

$(17.077)(10^{-5})$ P1 = 0.9298 + 0.00624 ***X***

$(3.514)(10^{-5})$ P2 = 0.5429 + 0.00624 ***X***

$(2.118)(10^{-5})$ P3 = 0.00624 ***X***

$(1.379)(10^{-5})$ P4 = 0.5429 + 0.00624 ***X***

$(17.077)(10^{-5})$ P5 = 0.9298 + 0.00624 ***X***

Solve for X in equation below

$$\frac{0.9298''+(0.00624''/ft)(X)}{(17.077x10^{-5})(''/lbs)}+\frac{0.5429''+(0.00624''/ft)(X)}{(3.514x10^{-5})(''/lbs)}+\frac{(0.00624''/ft)(X)}{(2.118x10^{-5})(''/lbs)}$$

$$=\frac{0.5429''-(0.00624''/ft)(X)}{(1.379x10^{-5})(''/lbs)}+\frac{0.7426''-(0.00624''/ft)(X)}{(17.077x10^{-5})(''/lbs)}$$

+ 36.54 **X**	-5444.8
+177.58 **X**	-15449.6
+294.62 **X**	0.0
+452.50 **X**	+39369.1
+ 36.54 **X**	+4348.5
(997.78 lbs/ft) **X** =	22823.2 (lbs)

X = 22.87 ft

Longitudinal Temperature Load at Bents

P1 = 5444.8 + 36.54(22.87)= 6281 lbs

P2 =15449.6+177.58(22.87)= 19511 lbs

P3 = 294.62(22.87)= 6738 lbs

P4 = 39369.1 - 452.5(22.87)= 29020 lbs

P5 = 4348.5 - 36.54(22.87)= 3513 lbs

1.3.6 Gross Moment of Inertia for Column and Pile Bents

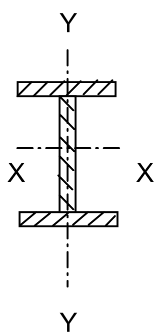
The moment of inertia shall be computed for any types, sizes, and number of piles to be used that are not given in these tables.

Concrete Columns

f'c = 3000 psi	n = 10	Ec = 3,300,000 psi
f'c = 4000 psi	n = 8	Ec = 3,800,000 psi
f'c = 5000 psi	n = 6	Ec = 4,300,000 psi

Gross Moment of Inertia – I(in^4)							
Col.Dia. (Ft.)	Number of Columns						
	1	2	3	4	5	6	7
2.5	39,760	79,520	119,280	159,040	198,800	238,560	278,320
3.0	82,448	164,896	247,344	329,792	412,240	494,688	577,136
3.5	152,745	305,490	458,235	610,980	763,725	916,470	1,069,215
4.0	260,576	521,152	781,728	1,042,304	1,302,880	1,563,456	1,824,032
4.5	417,393	834,786	1,252,179	1,669,572	2,086,965	2,504,358	2,921,751

Steel Pile Es = 29,000,000 psi



Number of Piles	Ixx(in^4)			Iyy(in^4)		
	HP10x42	HP12x53	HP14x73	HP10x42	HP12x53	HP14x73
1	210	393	729	71.7	127	261
4	840	1,572	2,916	287	508	1,044
5	1,050	1,965	3,645	359	635	1,305
6	1,260	2,358	4,374	430	762	1,566
7	1,470	2,751	5,103	502	889	1,827
8	1,680	3,144	5,832	574	1,016	2,088
9	1,890	3,537	6,561	645	1,143	2,349
10	2,100	3,930	7,290	717	1,270	2,610
11	2,310	4,323	8,019	789	1,397	2,871
12	2,520	4,716	8,748	860	1,524	3,132
13	2,730	5,109	9,477	932	1,651	3,393
14	2,940	5,502	10,206	1,004	1,778	3,654

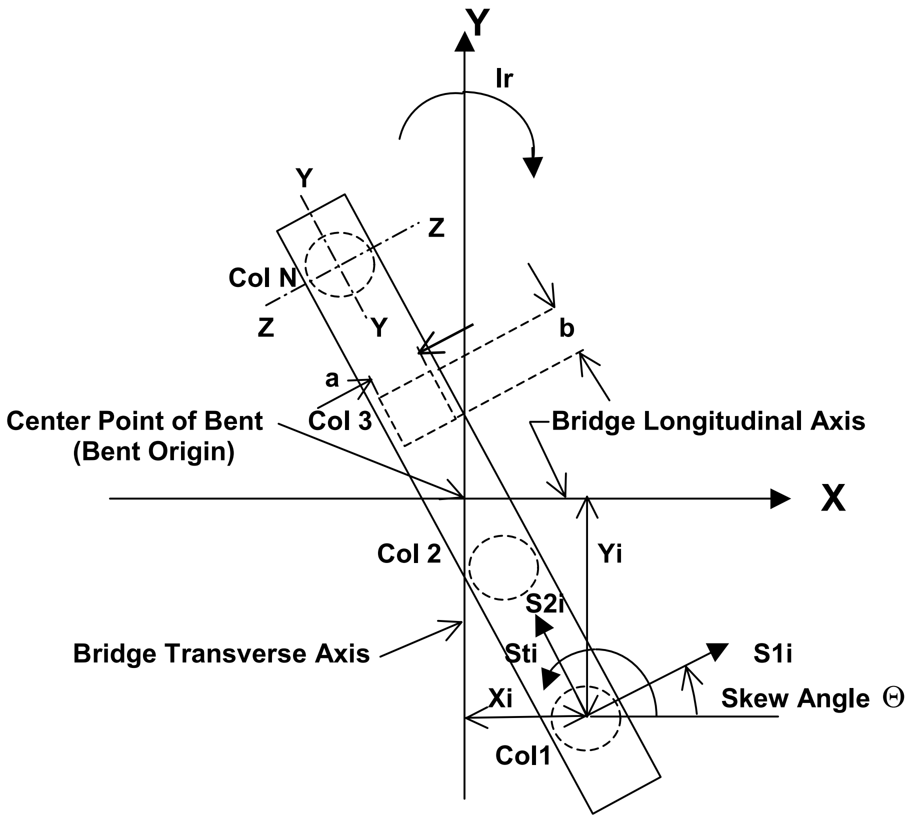
Alternate Pile

Gross moment of Inertia - I (in^4)	E' (*) f'c= 3 ksi	E' (*) f'c=4 ksi	E' (*) f'c=5 ksi
14" C.I.P. Pile: I = 1886	E' = 5515 ksi	E' = 5970 ksi	E' = 6430 ksi
20" C.I.P. Pile: I = 7854	E' = 5927 ksi	E' = 6376 ksi	E' = 6825 ksi
24" C.I.P. Pile: I = 16286	E' = 6365 ksi	E' = 6805 ksi	E' = 7245 ksi

(*)To account for the composite material properties as well as the geometric properties of the C.I.P. pile, apply the equation, $E'I = E_S I_S + E_C I_C$. Where E' is the equivalent modules of elasticity associated with the total moment of inertia, I. This will allow the longitudinal force distribution program to compute the correct stiffness for the bent containing the C.I.P. piles.

1.3.7 Longitudinal Bent Stiffness

Resultant Moment of Inertia



Terms

S1i = Stiffness of the i^{th} column normal to the bent (units of force per length)
S2i = Stiffness of the i^{th} column parallel to the bent
Sti = Tortional stiffness of the i^{th} column
Θ = Skew angle (positive in counterclockwise direction)
$\mathbf{X_i}$ = Coordinate distance from the bent origin to the i^{th} column considered along the bridge longitudinal axis (+/-)
$\mathbf{Y_i}$ = Coordinate distance from the bent origin to the i^{th} column considered along the bridge transverse axis (+/-)
$\mathbf{e_i}$ = $-Y_i \cos(\Theta) + X_i \sin(\Theta)$
(*)$\mathbf{f_i}$ = $X_i \cos(\Theta) + Y_i \sin(\Theta)$
N = total number of columns

(*) $\mathbf{f_i}$ =0 in most cases when the direction of column principal axis Y-Y is the same as the center line of the bent.

Moment of Inertia for a Skewed Bent

In the distribution of loads in the bridge longitudinal direction, the stiffnesses in the bridge longitudinal and transverse directions are coupled for a skewed bent. Therefore, the bent will experience a deflection in the bridge longitudinal direction and the bridge transverse direction simultaneously. To account for this coupling effect, Matrix Structural Analysis is used here to determine the bent stiffness matrix which consists of stiffnesses S1, S2 and St of all individual columns.

To simplify this analysis, use the following procedure.

Moment of inertias of the individual column

I_Y = Column moment of inertia parallel to the bent (in^4)

$$I_y = \frac{\pi r^4}{4} \text{ (circular)} \qquad \text{or} \qquad I_y = \frac{ba^3}{12} \text{ (rect)}$$

I_Z = Column moment of inertia perpendicular to the bent (in^4)

$$I_Z = \frac{\pi r^4}{4} \text{ (circular)} \qquad \text{or} \qquad I_Z = \frac{ab^3}{12} \text{ (rect)}$$

J = Polar moment of inertia (in^4)

$$J = \frac{\pi r^4}{2} \text{ (circular)} \qquad \text{or} \qquad J = \frac{ba(b^2 + a^2)}{12} \text{ (rect)}$$

Where

r is the radius of a circular column (in).
a and **b** are the widths normal and parallel to the bent respectively (in).

Stiffnesses of the individual column

After calculating the inertias of the columns the stiffnesses of the bent can be figured from the following.

S1 = Stiffness of the individual column normal to the bent (kip/in).

$$S1 = \frac{3EI_Y}{L^3} \text{ (kip/in.)}$$

Where

E is the Modulous of Elasticity of the column (ksi).
I_Y is the Moment of Inertia of the column (in^4).
L is the unsupported length from the top of the beam to the bottom of the column (in).

Stiffnesses of the individual column (Cont.)

S2 = Stiffness of the individual column parallel to the bent (kip/in).

$$S2 = \frac{12EI_Z}{L^3} \text{ (kip/in)}$$

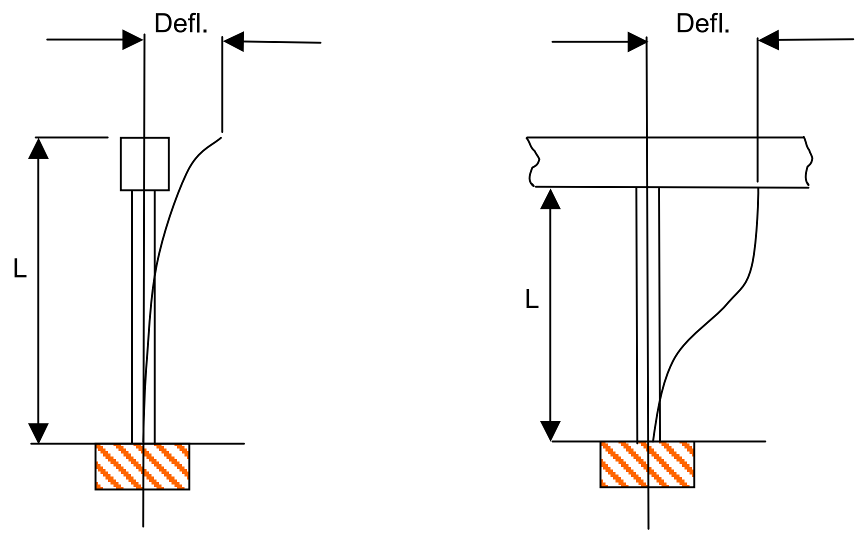
Where

E is the Modulous of Elasticity of the column (ksi).

I $_Z$ is the Moment of Inertia of the column (in^4).

L is the unsupported length from the bottom of the beam to the bottom of the column (in).

The difference in the stiffness for each direction comes from the way in which the column frames into the beam in each direction. In the direction normal to the bent, the column is considered fixed at the bottom and allowed to freely deflect and rotate at the top. In the direction parallel to the bent however, the column is fixed at the bottom, and able to deflect at the top but not to rotate. Notice also that the unsupported lengths "L" are different in each direction. (See Figure) The above equations may then be derived using the slope deflection method.



Normal to Bent **Parallel to Bent**

St = Torsional stiffness of the individual column.

$$St = \frac{GJ}{L} \text{ (kip-in/rad)}$$

Where **G** is the Shear Modulous of the column (ksi).

J is the Polar Moment of Inertia (Torsional Constant) of the Column (in^4).

L is the unsupported length (in). Use the average of the two lengths calculated for S1 and S2.

Stiffness Coefficients of Bent

$$\textbf{C1} = \sum_{i=1}^{N} \{\cos^2(\theta) S1i + \sin^2(\theta) S2i\}$$

$$\textbf{C2} = \sum_{i=1}^{N} \{e_i \cos(\theta) S1i - f_i \sin(\theta)\ S2i)\}$$

$$\textbf{C3} = \sum_{i=1}^{N} \{\cos(\theta) \sin(\theta)\ (S1i - S2i)\}$$

$$\textbf{C4} = \sum_{i=1}^{N} \{Sti + e_i^{\ 2}\ S1i + f_i^{\ 2}\ S2i\}$$

$$\textbf{C5} = \sum_{i=1}^{N} \{e_i \sin(\theta)\ S1i + f_i \cos(\theta)\ S2i\}$$

$$\textbf{C6} = \sum_{i=1}^{N} \{\sin^2(\theta)\ S1i + \cos^2(\theta)\ S2i\}$$

Resultant Longitudinal Stiffness

$$\textbf{Sr} = A3 - \frac{A2}{A1}\ (kip/in)$$

Where

A1 = $(C4)(C6)-(C5)^2$

A2 = $(C2)^2(C6)-2(C2)(C3)(C5)+(C3)^2(C4)$

A3 = C1

Resultant Moment of Inertia

Thus the resultant moment of inertia for the bent about the bridge longitudinal axis can be expressed as

$$\textbf{Ir} = \frac{L^3}{3E}[Sr]\ (in^4)$$

Where

L is the unsupported length (in) for the bent. Use the average of the two lengths calculated for S1 and S2.
E is the Modulous of Elasticity of the concrete (ksi).
Sr is as shown above (kip/in).